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EFFECT OF INCOME DISTRIBUTION ON THE ENVIRONMENTAL KUZNETS CURVE

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Abstract. This paper augments the model of Andreoni and Levinson by analyzing the effect of income distribution on the inverted U-shaped relationship between some forms of pollution and income, the so-called 'environmental Kuznets curve'. In a context in which pollution abatement technology shows increasing returns to scale and an inverse U-shaped pollution–income path is present, this study demonstrates the existence of a majority voting equilibrium, and concludes that the inverted U-shaped relationship is between median income and environmental degradation rather than between per capita income and environmental degradation. Our results suggest that an increase in equality in income distribution improves environmental quality and social efficiency. The implication of the model for the empirical estimation of environmental Kuznets curves is examined using a panel data set of 36 countries over a 20-year period. Estimation results using different models show that income distribution might be an important factor in the empirical estimation of these curves.

1. INTRODUCTION

Economic growth can have both negative environmental consequences, through scalar increases in economic activity, and positive environmental consequences, through increases in income that lead to the adoption of cleaner production methods (see Antweiler *et al.*, 2001). For the past decade or so, the research on the empirically demonstrated inverted U-shaped pattern (the so-called environmental Kuznets curve (EKC)) in which environmental degradation rises and then declines relative to per capita income has been very active. Empirical studies have documented significant evidence of the existence of the EKC (see e.g. Selden and Song, 1994; Grossman and Krueger, 1995; Halkos, 2003, among others) between economic growth and at least some forms of pollution, such as urban air pollutants, including ambient sulphur dioxide (SO_2), oxides of nitrogen (NO_x) and suspended particulates (SPM).

Environmental Kuznets curve theory has generated considerable theoretical speculation. Past work has demonstrated that an inverted U-shaped path may occur in a variety of model structures (e.g. Lopez, 1993; Selden and Song, 1995; Stokey, 1998; Kelly, 2003; Chimeli and Braden, 2005). In contrast to complex model structures that require strong technical assumptions or special constraints on preferences, a simple structure with a specific constraint on technology is

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proposed by Andreoni and Levinson (2001). They show that increasing-returns-to-scale abatement technology is a necessary and sufficient condition for the inverted U-shaped pollution path to exist in their model. They also present empirical support for increasing returns to abate some common air pollutants, which is further confirmed by Managi's (2006) empirical study.

The present paper examines the effect of income distribution on the Kuznets relationship between economic growth and environmental degradation. It seems plausible that income distribution affects social preferences and, therefore, that social choice drives at least some of the shift to a cleaner environment. However, research on this effect is scarce. An empirical study by Torras and Boyce (1998) finds ambiguous evidence that income inequality reduces environmental quality. Eriksson and Persson (2003) augment Stokey's (1998) model and suggest that the impact of income distribution on aggregate pollution depends on the degree of democracy. In a recent paper that is more closely related to the current study, Bousquet and Favard (2005) propose a simple environmental model in which income inequalities follow a bell curve and show that an EKC is not necessarily a bell curve in this context. In contrast to Bousquet and Favard (2005), however, our paper argues that the effect of income distribution on an EKC lies in a variant of the simple model proposed by Andreoni and Levinson (2001). Using an analytical framework similar to that of Andreoni and Levinson (2001), but with tax determined through majority voting, the present study derives the conditions under which income distribution and its dynamics play an important role in determining the specific pattern of an EKC, and demonstrates the existence of a majority voting equilibrium (MVE) in which the decisive voter (the individual with a median income) chooses the final tax rate (or the fixed amount of tax per capita). Therefore, the inverted U-shaped relationship is between median income and environmental degradation rather than between per capita income and environmental degradation.

The findings have important implications for policy issues from the point of view of social efficiency. In a context in which the pollution–income path shows an inverted U-shaped pattern, a more skewed income distribution will push back the appearance of environmentally friendly economic growth (the down-turned portion of an EKC), whereas a less skewed pattern of income distribution will help to produce a positive environment–income relation. Taking economic growth as a given, more equal distribution of income in society improves social efficiency in terms of consumption and environmental quality. However, because income distribution in the real world is generally skewed leftward, the allocation of resources is not socially efficient.

The results also have important implications for the empirical estimation of the EKC. The explanatory variables common to all econometric studies of the pollution–income relationship are real per capita GDP and its square. If the effect of income distribution on the pollution–income path is not considered, these estimation results might be biased. The empirical component of this paper estimates the EKC using a panel data set that includes 36 countries over a 20-year period. The effect of income distribution is taken into consideration in the estimation models.

The remainder of the paper is structured as follows. Section 2 lays out the benchmark model with only one person in the economy. Section 3 generalizes the model to the multiple-consumer case and derives the main analytical results. Section 4 discusses the implication of the model for social efficiency and policy issues. Section 5 presents some empirical results of the estimation of EKC in light of the theoretical findings. Section 6 concludes the paper.

2. THE BENCHMARK MODEL

The basic analytical framework used in the present paper is a variant of the model in Andreoni and Levinson's (2001). In this structure, the observed inverted U-shaped pattern does not require dynamics, predetermined patterns of economic growth, multiple equilibriums, released constraints, bundled commodities or irreversible pollution. Compared to other models that incorporate these features, this framework is relatively simple, yet it encompasses many of the other explanations of the pollution–income relationship. The key difference between Andreoni and Levinson's (2001) model and the model that is proposed in this paper is that in the present model, the inverted U-shaped relationship is investigated in an environment with tax, and the tax level is determined through majority voting. In addition, unlike the case of the model of Andreoni and Levinson (2001), which assumes equal endowment of wealth for each consumer, in the present model, consumers are endowed with different incomes.

Consider a simple case in which only one consumer is in the economy. This implies that no environmental externality exists and any solution is Pareto efficient. The preference of this single consumer is given by $U = U(C, P)$, where C is the consumption of a private good, and $Uc > 0$. The consumer gets negative utility (or disutility) from pollution, which is denoted as P; that is, $Up < 0$. Assume that U is well behaved in that it is quasi-concave in C and $-P$, and is second-order continuously differentiable. There are no special restrictions on preference other than the usual ones in this model. Consider a simple functional form of $U(C, P)$:

$$U = U(C, P) = C - \lambda P. \tag{1}$$

In equation 1, utility is defined as additively separable in consumption and pollution. This is a stylized form of utility function (see e.g. Selden and Song, 1995; Stokey, 1998).

Pollution is caused by consumption, and can be alleviated by allocating resources to pollution abatement. The functional form of pollution is defined as $P = P(C, E)$, where E denotes resources that are devoted to pollution abatement. Pollution increases with consumption and decreases with abatement effort; that is, $Pc > 0$ and $P_E < 0$. The production function of pollution abatement is a function of consumption and the resources allocated to abatement, and increases on both abatement inputs and gross pollution caused by consumption before abatement. The production technology shows increasing returns to scale. In other words, the more gross pollution there is before abatement, the less costly it is to abate one unit of that pollution. This feature of increasing-returns-to-scale pollution abatement technology is the key assumption of an inverse-U-shaped pollution–income path in the model of Andreoni and Levinson (2001), which is supported by empirical evidence (see e.g. Managi, 2006). Assume that the pollution function takes the following form:

$$P = P(C, E) = C - C^{\alpha} E^{\beta}. \tag{2}$$

The first term on the right-hand side of equation 2 gives the effect of consumption on pollution. The second term is the production function of pollution abatement, $A(C, E) = C^{\alpha}E^{\beta}$. It is a standard Cobb–Douglas concave production function with increasing-returns-to-scale technology. Here, $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and $\alpha + \beta > 1$ (increasing-returns-to-scale technology).

The single consumer in the economy is endowed with W, which is limited in quantity and is the only source of the consumer's income. This endowment of resources can be spent on either consumption, C, or pollution abatement, E.[1] Normalize the relative prices of C and E to 1 and the consumer's budget constraint is given by:

$$C + E = W. \tag{3}$$

The allocation of resources to pollution abatement is realized through collecting a tax. Assume that the tax is proportional to the consumer's income (see the Appendix for a head-tax regime). In this simple one-person case, the single consumer chooses the tax rate that is optimal to him or her and to society. Denote τ as the tax rate, and

$$E = E(\tau) = \tau W \tag{4}$$

$$C = C(\tau) = (1 - \tau)W. \tag{5}$$

Solve the single consumer's optimization problem, which is to maximize his or her utility, $U = U(C, P)$, by choosing τ, subject to equations 2, 3 and 4. This is equivalent to maximizing $\lambda(1 - \tau)^{\alpha}\tau^{\beta} + (1 - \lambda)(1 - \tau)W^{1-\alpha-\beta}$ by choosing τ. Equation 6 can be derived from the first-order condition:

$$F(\tau, W) = \lambda(1-\tau)^{\alpha}\tau^{\beta}\left(\frac{\beta}{\tau} - \frac{\alpha}{1-\tau}\right) - (1-\lambda)W^{1-\alpha-\beta} = 0. \tag{6}$$

The optimal tax rate, τ^*, is the solution to equation 6, and has the following properties.

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[1] For simplicity, this model excludes forms of production other than pollution abatement. The economic growth is exogenous and is reflected by increasing W from outside sources.

PROPERTY A. τ^* is the maximum solution to the consumer's problem, and the consumer's utility is single peaked with respect to τ.

PROOF: Check the second-order condition:

$$F_\tau(\tau, W) = \lambda(1-\tau)^\alpha \tau^\beta \left[\frac{\beta(\beta-1)}{\tau^2} + \frac{\alpha(\alpha-1)}{(1-\tau)^2} - \frac{2\alpha\beta}{\tau(1-\tau)}\right] < 0. \tag{7}$$

Equation 7 holds because both α and β are less than 1. It is easy to see from equation 7 that for any given value of λ, there is only one solution for equation 6. Therefore, Property A holds.

PROPERTY B.[2] When $\lambda \neq 1$, the optimal tax rate, τ^*, increases monotonically or decreases with respect to the consumer's endowment (income), W.

PROOF:

$$\frac{\partial \tau^*}{\partial W} = -\frac{F_W}{F_\tau}. \tag{8}$$

$\frac{\partial \tau^*}{\partial W}$ has the sign of F_W; that is, the sign of $(\alpha+\beta-1)(1-\lambda)$. Given that $\alpha+\beta > 1$, $\frac{\partial \tau^*}{\partial W}$ is positive or negative when $\lambda < 1$ or $\lambda > 1$. Therefore, Property B holds.

It can be shown that the optimal pollution level, P^*, that corresponds to τ^*, is concave and shows an inverted U-shaped pattern with respect to W (for the detailed proof, see Andreoni and Levinson, 2001), as depicted in Figure 1.

Denote the optimal tax rate corresponding to the turning point (W^{**}) as τ^{**}. If $\frac{\partial \tau^*}{\partial W} > 0$, then τ^* increases as W increases, and the optimal pollution level P^* increases first; then, after the turning point, W^{**} and τ^{**}, it decreases. If $\frac{\partial \tau^*}{\partial W} < 0$, τ^* decreases as W increases, and the optimal pollution level P^* increases first, then, after the turning point, it decreases.

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[2] When $\lambda = 1$, the optimal tax rate is constant. This is a trivial case and does not affect the main analytical results. See Section 3. The proofs of both Property A and Property B benefit greatly from an anonymous referee and we thank the anonymous referee for their effort.

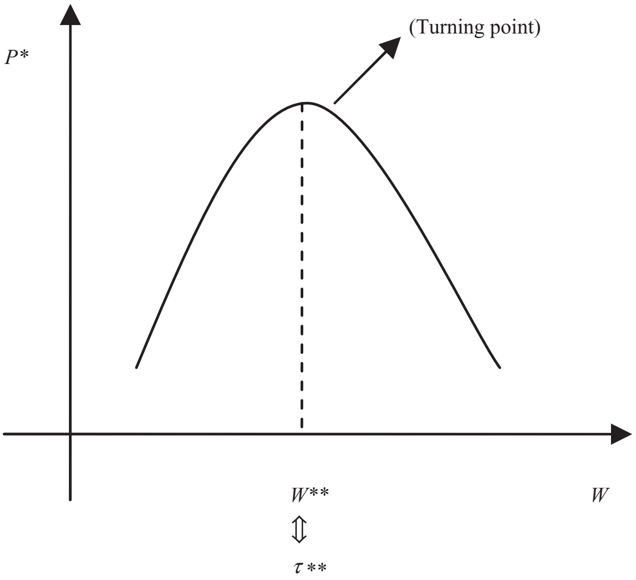

Figure 1. Optimal pollution–income path

It is clear from theorem 1 of Andreoni and Levinson (2001) that for a general form of the utility function, the inverse-U-shaped pollution-income path remains (for the detailed proof, see Andreoni and Levinson, 2001).[3]

3. MULTIPLE CONSUMERS AND THE EXISTENCE OF A MAJORITY VOTING EQUILIBRIUM

Now we generalize the model to the multiple-consumer case. Assume that there are N consumers in the economy. Consumers have the same utility function, but are endowed with different levels of income. Consumers vote on a tax rate, and the final tax rate is chosen through majority rule. For individual i with endowment W_i, the optimal tax rate can be derived by solving the following maximization problem:

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[3] Theorem 1 in Andreoni and Levinson (2001) shows the following (slightly revised from its original form).

Assume that the utility function $U(C, P)$ is quasi-concave in C and $-P$, and that C and $-P$ are both normal goods. Then, if there exists a value θ such that

$$\lim_{C \to W} R(C) \equiv \frac{\partial U(C,0)/\partial C}{\partial U(C,0)/\partial P} \geq \theta > -\infty$$

and a pollution abatement function, $A = A(C, E) = A(C, M - C)$, is homogeneous of degree $k > 1$ in the variables (C, E) and concave in the variable C (for the function $A(C, M - C) \equiv a(C)$), where M is the resource endowment for consumers, $P(0, x) = 0$, and $P(x, 0) = x$ for all x, then for any combination of utility and abatement technology that yields positive pollution for some level of income, optimal pollution will eventually decline to zero for a sufficiently large income.

$$\max_{\tau_i} U_i = U(C_i, P) = C_i - \lambda P, \tag{9}$$

subject to

$$P = C - C^{\alpha} E^{\beta} \tag{2}$$

$$C = C_i + C_{-i} = (1 - \tau_i)(W_i + W_{-i}) \tag{10}$$

$$E = \tau_i (W_i + W_{-i}), \tag{11}$$

where $C_{-i} = \sum_{j \neq i} C_j$ is defined as the sum of the consumption of all other consumers and $W_{-i} = \sum_{j \neq i} W_j$ is defined in a similar way. Consumer *i* treats W_{-i} as given. His or her choice of τ_i (if it becomes the final tax rate chosen by society) will affect the utility of other consumers even though he or she tries to maximize only his or her own utility; that is, externality is introduced in the multiple-consumer case.

The first-order condition is given by:

$$\begin{aligned} \frac{\partial U_i}{\partial \tau_i} &= -W_i + \lambda (W_i + W_{-i}) - \lambda (W_i + W_{-i})^{\alpha+\beta} \alpha (1 - \tau_i)^{\alpha-1} \tau_i^{\beta} \\ &\quad + \lambda (W_i + W_{-i})^{\alpha+\beta} \beta (1 - \tau_i)^{\alpha} \tau_i^{\beta-1} = 0. \end{aligned} \tag{12}$$

The second-order condition is:

$$\begin{aligned} \frac{\partial^2 U_i}{\partial \tau_i^2} &= \lambda (W_i + W_{-i})^{\alpha+\beta} \alpha (\alpha - 1)(1 - \tau_i)^{\alpha-2} \tau_i^{\beta} - \lambda (W_i + W_{-i})^{\alpha+\beta} \alpha (1 - \tau_i)^{\alpha-1} \beta \tau_i^{\beta-1} \\ &\quad - \lambda (W_i + W_{-i})^{\alpha+\beta} \beta (1 - \tau_i)^{\alpha-1} \alpha \tau_i^{\beta-1} \\ &\quad - \lambda (W_i + W_{-i})^{\alpha+\beta} \beta (1 - \tau_i)^{\alpha} (1 - \beta) \tau_i^{\beta-2} < 0. \end{aligned} \tag{13}$$

It is clear from equations 12 and 13 that the optimal tax rate for consumer *i*, τ_i^*, which is the solution to equation 12, is the only maximum solution for his or her optimization problem. Consumer *i*'s utility is single peaked with respect to tax rate τ_i. Property A in Section 2 still holds in this multiple-consumer case. To discover whether property B also holds in the multiple-consumer case, total differentiation of equation 12 yields:

$$\frac{\partial \tau_i^*}{\partial W_i} = \frac{[(\lambda - 1) W_i + \lambda W_{-i}](W_i + W_{-i})^{-1}(\alpha + \beta) - (\lambda - 1)}{\partial^2 U_i / \partial \tau_i^{*2}}. \tag{14}$$

When $\lambda > 1$, the numerator of equation 14 is greater than zero, and $\frac{\partial \tau_i^*}{\partial W_i} < 0$; when $\lambda \leq 1$, assume that N is large (there are many consumers in this economy), and assume that each individual's endowment is small relative

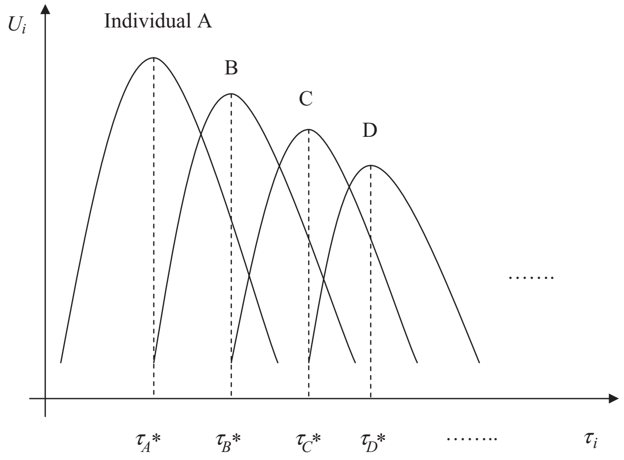

Figure 2. Optimal tax rates for individuals with different endowments

to the total endowment of all consumers. These assumptions imply that $\frac{W_{-i}}{W_i + W_{-i}}(\alpha+\beta) \approx \alpha+\beta$. The numerator of equation 14 is again greater than zero and $\frac{\partial \tau_i^*}{\partial W_i} < 0$.

The above discussion suggests that the optimal tax rate for each individual decreases monotonically with the individual's endowment (income). This analytical result requires only the weak assumption that N is sufficiently large, which is easily met in the real world. Therefore, the optimal choices of tax rates for individuals can be ordered according to their endowment (income). The higher an individual's endowment (income) is, the lower the individual's preferred tax rate.[4]

Figure 2 illustrates the above results. Assume that individuals (A, B, C, D ⋯) have different endowments. Specifically, $W_A > W_B > W_C > W_D \cdots$, and their optimal choices of tax rate are ordered in the opposite direction; that is, $\tau_A^* < \tau_B^* < \tau_C^* < \tau_D^* \cdots$. Each individual's utility is single peaked.

As the tax rate is the only choice variable in this context (single dimension of choice), and consumer preferences are single peaked, according to Black's theorem, an MVE exists[5]. Furthermore, because the optimal tax rate preferred by each consumer decreases monotonically with an increase in consumer

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[4] One may argue that W_{-i} is different for different individuals. However, under the assumptions that N is large and that W_i is small relative to W_{-i}, we can treat W_{-i} as a constant and assume that it has the same value in every individual's optimization problem. The situation can be approximated as follows: assume that consumer i does not have perfect information about W_{-i}. However, somehow he or she has credible information about the mean income, W_{mean}, and population size, N. Therefore, the value of W_{-i} is equal to $(N-1)W_{mean}$ for everyone.

[5] Black (1948) makes the following proposition about majority voting: if all voter preferences are single peaked in a single dimension, then the median ideal preference is the winner (MVE).

income, the consumer with a median income is the decisive voter. In other words, the optimal tax rate preferred by the individual with a median income is the final tax rate chosen by a society.

In this multiple-consumer case, the inverse-U-shaped pollution–income path remains (for a detailed proof, see Andreoni and Levinson, 2001). The new insight is that when there are many consumers with different income levels in the economy, and when majority rule is the way social decisions are made, the tax rate (and, hence, the pollution level) is determined by the individual with a median income. Therefore, the inverted U-shaped pattern of the pollution–income relationship is between median income and pollution level rather than between mean income and pollution level. This result does not depend on the assumption that the tax is proportional to a consumer's income. Similar analytical results can also be derived in a head-tax regime (see the Appendix).

4. MODEL IMPLICATIONS: SOCIAL EFFICIENCY AND POLICY ISSUES

The model generates important implications for social efficiency. It is now clear that income distribution and its variation play an important role in forming the specific inverted U-shaped pattern of the pollution–income path. To illustrate this effect, define m as the ratio of median income to mean income as follows:

$$m = \frac{W_{median}}{W_{mean}}. \tag{15}$$

If income distribution is normal (or symmetric at least), then median income will be the same as mean income; that is, $m = 1$. However, as income distribution is almost always skewed leftward in the real world, the median income is lower than the mean income ($W_{median} < W_{mean}$), and $m < 1$. The more skewed the income distribution is, the lower the value of m. Therefore, the m ratio is a measure of how skewed the income distribution is, and it is often used as a measure of income inequality.

Figure 3 illustrates different pollution–income paths given different values of m. Assume that curve 1 in Figure 3 shows the pollution–income (mean) path when income distribution is normal ($m = 1$) and does not change over time; that is, m is constant. Curve 2 in Figure 3 depicts the pollution–income (mean) path when income distribution is skewed to the left (m is constant over time and is equal to 0.75). Notice that when income distribution is skewed leftward, the emergence of environmentally friendly economic growth (turning point in terms of the mean income) is pushed back in the income horizon (in Fig. 3, $W_2^* > W_1^*$).

Now take the dynamics of income distribution into consideration. Suppose that as the median income increases from W_s to W_e, the m ratio increases uniformly from 0.75 to 1; that is, income distribution becomes less skewed over time. Curve 3 in Figure 3 shows the pollution–income path in this case. Similarly, curve 4 depicts the case when the m ratio decreases from 0.75 to 0.5. It is clear that when income distribution becomes less skewed (income distribution is

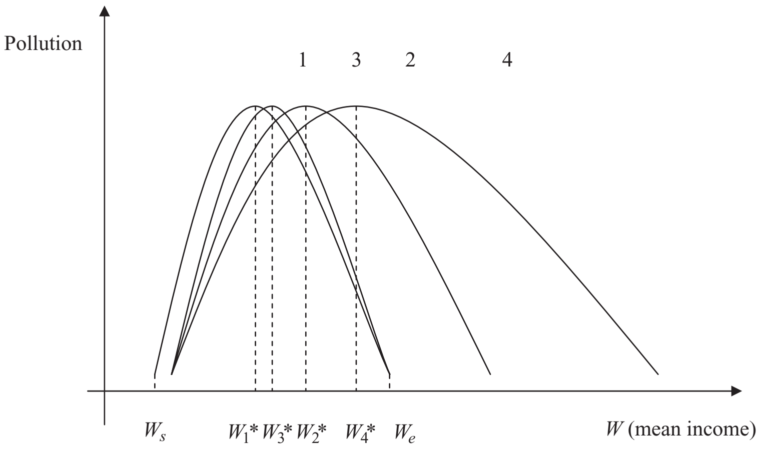

Figure 3. Different pollution–income paths. Curve 1: pollution–income path when m = 1. Curve 2: pollution–income path when m = 0.75. Curve 3: pollution–income path when income distribution becomes less skewed over time (m increases from 0.75 to 1 uniformly). Curve 4: pollution–income path when income distribution becomes more skewed over time (m decreases from 0.75 to 0.5 uniformly)

more equal), the turning point is pushed ahead (in Fig. 3, $W_2^* > W_3^*$). If income distribution becomes more skewed over time (income distribution becomes more unequal), then the value of *m* decreases and the emergence of a negative pollution–income relation is pushed back further (in Fig. 3, $W_4^* > W_2^*$).

The policy suggestion based on the above discussion is straightforward: as long as economic growth is measured in terms of per capita income and the growth rate is given,[6] mean-preserving reduction in the variance of income, where the median is less than the mean, shifts the EKC to the left; that is, reduces the level of income at which pollution peaks. Therefore, a public policy that aims to make income distribution less skewed will, among other gains, actually benefit the environment. This argument is further strengthened by examining analytically the model's implication for social efficiency. Assume that the central planner of a society tries to achieve social efficiency by maximizing the sum of the utility of all consumers:

$$\max_{\tau} U = \sum_{N} U_i(C_i, P) = \sum_{N} C_i - \lambda NP, \tag{16}$$

subject to

$$\sum_{N} C_i = (1-\tau)\sum_{N} W_i \tag{17}$$

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[6] Discussion of the sources of economic growth is outside the scope of the present paper.

$$P = (1-\tau)\sum_N W_i - \left[(1-\tau)\sum_N W_i\right]^{\alpha}\left[\tau\sum_N W_i\right]^{\beta}. \tag{18}$$

Solve for the first-order condition:

$$\frac{\partial U}{\partial \tau} = (\lambda N - 1)\sum_N W_i - \lambda\alpha N\left(\sum_N W_i\right)^{\alpha+\beta}(1-\tau)^{\alpha-1}\tau^{\beta} + \lambda\beta N\left(\sum_N W_i\right)^{\alpha+\beta}(1-\tau)^{\alpha}\,\tau^{\beta-1} = 0. \tag{19}$$

Comparing equation 19 with equation 12, it is obvious that the optimal τ^* chosen by the social planner is the same as that chosen by the decisive voter (the consumer with a median income) if and only if the following is true:

$$W_{median} = \frac{1}{N}\sum_N W = W_{mean}. \tag{20}$$

Therefore, the tax rate chosen through majority voting is socially optimal only when the median income is equal to the mean income. Given that income distribution in the real world is skewed, the resulting tax rate chosen by majority rule will not be Pareto efficient in the sense that there are potential redistributions that make everybody better off. From equation 14, it is clear that the MVE tax rate is higher than the socially efficient tax rate, which means overinvestment in pollution abatement. This result suggests that a less skewed income distribution benefits society as a whole (higher social efficiency) in terms of consumption and environmental quality.

It should be noted that the above analytical results depend on the model specification that tax is proportional to income. In a head-tax regime, the MVE tax level is not Pareto efficient, but it is lower than the socially efficient tax level, which implies underinvestment in pollution abatement (see the Appendix for a detailed discussion). The bottom line here is that when income distribution is skewed leftward, the resulting pollution abatement level (thus, the pollution level) determined by majority rule is not socially efficient.

5. IMPLICATION OF THE MODEL FOR THE EMPIRICAL ESTIMATION OF AN ENVIRONMENTAL KUZNETS CURVE: AN ILLUSTRATION

The model outlined in Section 3 suggests that income distribution needs to be considered and median income should be used in the estimation of an EKC. However, the explanatory variables common to almost all econometric studies of the economic growth–environment relationship are real per capita GDP and its square. Given that income distribution is usually skewed leftward in the real world, the existing estimation results of EKC might be biased. In this section, we conduct a simple exercise to illustrate the model implication on the empirical estimation of EKC by using median income instead of mean income in the regressions.

5.1. *Estimation model*

We adopt an uncomplicated estimation model, which is similar to the ones commonly used in the literature (see e.g. Selden and Song, 1994; Grossman and Krueger, 1995). The primary model specification is as follows:

$$p_{it} = \beta_{0i} + \beta_1 W_{it} + \beta_2 W_{it}^2 + \beta_3 D_{it} + \beta_4 D_{it}^2 + \beta_5 U_{it} + \beta_6 U_{it}^2 + \varepsilon_{it}, \tag{21}$$

where p is per capita emissions, W is median income of the country, D is population density, U is a measure of urbanization, i is a country index, t is a time index, β_{0i} is a country effect, and ε_{it} is a remaining error term with a mean zero and finite variance. The 'β's are the parameters to be estimated. Of particular importance are the sign and value of β_1 and β_2, as the 'turning point' of EKC is estimated as $-\beta_1/2\beta_2$, and it should be positive with a reasonable value. Population density is included as a control variable because sparsely populated countries may be less motivated to reduce per capita emissions at every level of income (Selden and Song, 1994). We also control for the effects of urbanization in the regressions because, for instance, urban residents are more concerned about air quality (Selden and Song, 1994) and pollution abatement might be more effective and cost-efficient in urban areas. Quadratic terms of the control variables are included in the regressions because their relations to the pollution might not be linear. In the present study, we use a one-way estimation model to analyze the panel data set instead of two-way analysis, following Grossman and Krueger (1995) and in contrast to Selden and Song (1994).[7]

5.2. *The data*

Unlike previous researchers, we use median income and its square as explanatory variables instead of mean income and its square in the regressions. The summary statistics of the variables used in the estimation are presented in Table 1.

Following previous work, we concentrate on two air pollutants, SO_2 and NO_x, because these pollutants have been the focus of considerable public policy

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[7] Selden and Song (1994) specify an error-components model in which

$$\varepsilon_{it} = c_i + v_t + u_{it}.$$

Although including the country effect (c_i) can be easily justified, a common and global time effect (v_t) is questionable. Unlike global warming pollutants (such as carbon dioxide), the sources of SO_2 and NO_X and their impacts on the environment are local. Indeed, Selden and Song (1994 p. 152) report that: 'the estimates (of period effects) are typically not significantly different from zero in the preferred fixed-effects models'. It seems that a one-way analysis is more appropriate. Grossman and Krueger (1995) use a one-way analysis instead of a two-way analysis.

Table 1. Summary statistics of variables

	Sample mean	Standard deviation	Maximum	Minimum	Number of observations
Sulfur dioxide (SO_2) (100 kg per capita)	0.63	0.57	3.24	0.04	141
Oxides of nitrogen (NO_x) (100 kg per capita)	0.38	0.22	1.10	0.03	137
Log-Median income (in 2000 US dollars)	9.44	0.54	10.19	7.87	112
Log-Mean income (in 2000 US dollars)	9.57	0.51	10.37	8.17	130
m ratio (= median income/mean income)	0.87	0.05	0.95	0.68	122
Gini coefficient	30.50	6.47	48	20	106
Population density (100 persons/ km^2)	1.14	0.99	4.57	0.02	141
Infant mortality rate (per 1000 live births)	13.46	14.20	103	4	119
Urban population (% of total)	68.80	13.10	97	34	123
Life expectancy at birth (years)	73.98	3.31	61	79	122

	Distribution of sample by income level		
	Low income	Middle income	High income
Number of countries in the sample, by level of development	0	16	20

Income classification based on the World Development Report.

attention, and have been included in most of the published empirical work on the estimation of EKC.[8] The source for country-level per capita emission data is GEMS aggregate emissions data (divided by country population), as in Selden and Song (1994). These data are obtained from various issues of *World Resources*. The data set includes 36 countries that reported emission data at least twice between 1980 and 1999. For each country in the sample, annual emission data in the years 1980, 1985, 1990, 1993, and 1996 are gathered, respectively.[9] These years are chosen because countries only report emission data every several years and the available data are clustered around these 5 years. The data set is an unbalanced panel (3.9 observations per country, on average). Both per capita SO_2 and per capita NO_x vary widely across countries, from 4.26 to 323.86 kg per capita in the case of SO_2 and from 2.77 to 109.89 kg per capita in the case of NO_x (Table 1).

Regarding the income data, one important question that needs to be dealt with is how to obtain a measurement of median income. In some countries, such as the United States, it is possible to obtain data about median income directly,

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[8] SO_2 is studied by Selden and Song (1994) and Grossman and Krueger (1995), among others. NO_x is studied by Selden and Song (1994) and Panayotou (1993), among others.

[9] These 5 years are years in which emission data are reported during the 1980–1999 period. If a country does not report emission data in a specific year, for example, in 1985, we use the data from the year that is closest to that year, for example, 1984 or 1986, if the data are available.

but this kind of information is generally not available in other countries. We address this problem by calculating an approximate measure of the median income in a country with publicly available information. First, we obtain data about the mean income (real per capita GDP) for country i at time t directly from the real GDP per capita series on real per capita GDP (in 2000 US dollars) drawn from the Penn World Table version 6.2 in Summers and Heston (1991). The per capita GDP data are then multiplied by a ratio m_{it} to obtain the median income, where m_{it} is defined as the ratio of the median income to the mean income in equation 5.

We construct an approximation of the ratio, m_{it}, by parsing the income distribution statistics from the World Development Report. The World Development Report provides information about the percentage of income allocated to the poorest to richest 20% of people in each country in some years of the sampling period. Under the assumption that income distribution in the group of the middle 20% is normal or close to normal, the percentage of income distributed to this group of people multiplied by five is a good approximate measurement of m_{it}. The mean value of m_{it} is 0.87 in the sample, which indicates that income distribution is generally skewed leftward. Inspection of the data reveals that income distribution varies significantly across countries, with a minimum m ratio of 0.68 and maximum m ratio of 0.95 (Table 1). The average median income is \$US14 196.35 (in 2000 US dollars), with the lowest income at \$US2609.75 and the highest at \$ US26 614.82.[10] Because of data constraints, low-income countries are not included in the data set. Among the 36 countries that are included in the sample, 16 are middle-income countries and 20 are high-income countries (World Bank classification). Of these countries, 27 are OECD countries.

The population density (100 persons per square km) data in different countries over time are obtained from the source of data for the estimation of the real per capita GDP measure. The information about urbanization is obtained from the online database of World Development Indicators compiled by the World Bank Group.

5.3. *Estimation results*

Table 2 presents the estimation results for SO_2 and NO_x, respectively, using pooled cross-section, feasible general least square (GLS) and random (country) effects estimation models. White heteroskedasticity consistent standard errors are reported in parenthesis for all the estimation models unless specified otherwise. Generally speaking, estimates of the main parameters of interest, β_1 and β_2, both have the expected signs and are statistically different from zero in most of the estimation models except the cross-section model for NO_x. The estimate of the turning point, $-\beta_1/2\beta_2$, is a reasonably low number. Both pollutants studied exhibit a meaningful Kuznets relationship with median income.

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[10] The natural logarithm of the median income is used in the regressions.

Table 2. Estimation results for sulfur dioxide (SO_2) and nitrogen (NO_x)

	Dependant variable: SO_2			Dependant variable: NO_x		
	Cross-section	Feasible GLS	Random effects	Cross-section	Feasible GLS	Random effects
	(1)	(2)	(3)	(4)	(5)	(6)
Log-median income (β_1)	11.46*** (1.86)	9.28*** (1.48)	11.72*** (2.40)	1.29 (0.86)	1.71*** (0.66)	3.27*** (0.96)
Log-Median income squared (β_2)	–0.65*** (0.10)	–0.53*** (0.08)	–0.66*** (0.13)	–0.07 (0.05)	–0.09** (0.04)	–0.17*** (0.05)
Population density	–0.25** (0.12)	–0.17*** (0.06)	–0.22 (0.22)	–0.26*** (0.04)	–0.28*** (0.03)	–0.19** (0.09)
Population density squared	0.03 (0.02)	0.02 (0.01)	0.02 (0.04)	0.05*** (0.01)	0.05*** (0.01)	0.03 (0.02)
Urbanization	0.05** (0.02)	0.06*** (0.02)	–0.32E–02 (0.03)	0.01 (0.01)	0.25E-02 (0.62E-02)	–0.01 (0.01)
Urbanization squared	–0.25E-03 (0.16E-03)	–0.28E-03** (0.11E-03)	0.89E-04 (0.21E-03)	–0.65E-06 (0.61E-04)	–0.38E-04 (0.45E-04)	0.61E-04 (0.80E-04)
Constant	–51.83*** (8.47)	–41.87*** (6.83)	–51.53*** (10.63)	–6.24 (3.95)	–7.99*** (3.05)	–14.87*** (4.31)
Turning point (in 2000 US dollars)	6737.10	6340.53	7178.09	–	13359.73	13406.36
Fixed versus random effects (Hausman)	–	–	2.94	–	–	5.29
(p-value)	–	–	(0.71)	–	–	(0.38)
R^2	0.43	–	0.38	0.56	–	0.45
Number of observations	100	100	100	97	97	97

'Log' means natural logarithm of variable values. ***, ** and * indicate that a coefficient estimate is significantly different from zero at the 1, 5 and 10% level, respectively. White heteroskedasticity-consistent standard errors are in parentheses unless specified otherwise. GLS, general least squares.

The simple cross-section estimation model does not take advantage of the nature of the panel dataset and ignores the potential different variance of the data for each of the panels. The feasible GLS model corrects for this potential source of heteroskedasticity across panels (countries) and generates more efficient estimation results (Table 2).

The results of the Lagrange Multiplier test suggest that pooled models might be inappropriate and might yield biased and inefficient coefficient estimates. Hausman's test is used to choose between a fixed-effects model and a random-effects model. This test argues in favour of a random-effects model for both SO_2 and NO_x (Table 2). The coefficients estimates of fixed-effects models are generally insignificant and, therefore, they are not reported in Table 2.

Both feasible GLS and random effects estimation models generate highly significant coefficient estimates of β_1 and β_2 (at the 1% significance level). The estimated turning points from the most favoured estimation models (random-effects model, in 2000 US dollars) are: SO_2, $US7178.09 (regression 3, Table 2), and NO_x, $US13 406.36 (regression 6, Table 2). The economic meaning of the turning point here is that when a country's median income approaches the turning point, a negative income–emission relationship appears. Because income distribution and its dynamics are different in different countries over time, one should expect to observe different turning points across countries if one looks at the relationship between pollution and per capita income in each country.[11] Table 3 illustrates the different mean income levels that correspond to the appearance of the environmentally friendly economic growth for some of the countries in the sample. We use the data on the m ratio from the latest year of a country in the sample and calculate the corresponding turning point in terms of mean income for some of the countries in the sample. Obviously this table is for illustration purposes only, as each country will have a different dynamic path of income distribution, which, in itself, is a complicated process. Because the income distribution data are quite irregular in the sampling period, it is not possible to observe the dynamic change in income distribution within a country, and it is beyond the scope of this paper to describe the exact pattern of the change in income distribution for each country.

Our findings are robust to controlling for the effects of population density and urbanization in the regressions. Both population density and urbanization show some nonlinear relationship with pollution. However, the coefficient estimates on the two variables are generally insignificant.

5.4. *Further robustness checks*

The model developed in Section 3 is based on democratic institutions, an assumption that is more likely to be true in OECD countries. We run the same regressions (random-effects model) for the OECD subsample, which includes only OECD countries. The estimation results are reported in Table 4 (regres-

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[11] Our data show significant variations in income distribution among countries but the documentation of the change in income distribution is insufficient because of data availability.

Table 3. Illustration of the different turning points (mean income-pollution relationship)

Country	*m* ratio (corresponding to the latest year in the sample)	Turning point (mean income, in 2000 US dollars) Sulphur dioxide	Turning point (mean income, in 2000 US dollars) Oxides of nitrogen	Latest year in the sample
Albania	0.87	8250.68	15 409.61	1990
Austria	0.93	7718.38	14 415.44	1996
Belarus	0.93	7718.38	14 415.44	1996
Belgium	0.92	7802.27	14 572.13	1996
Bulgaria	0.90	7975.66	14 895.96	1990
Canada	0.86	8346.62	15 588.79	1996
Czech Republic	0.85	8444.81	15 772.19	1996
Denmark	0.92	7802.27	14 572.13	1996
Finland	0.88	8156.92	15 234.50	1996
France	0.86	8346.62	15 588.79	1996
Greece	0.85	8444.81	15 772.19	1996
Hungary	0.85	8444.81	15 772.19	1996
Ireland	0.82	8753.77	16 349.22	1996
Italy	0.84	8545.35	15 959.95	1990
Japan	0.88	8156.92	15 234.50	1990
Netherlands	0.84	8545.35	15 959.95	1996
Norway	0.90	7975.66	14 895.96	1996
Poland	0.89	8065.27	15 063.33	1996
Portugal	0.80	8972.61	16 757.95	1990
Romania	0.88	8156.92	15 234.50	1990
Sweden	0.91	7888.01	14 732.26	1996
Switzerland	0.87	8250.68	15 409.61	1996
Turkey	0.74	9700.12	18 116.70	1990
Ukraine	0.91	7888.01	14 732.26	1996
UK	0.82	8753.77	16 349.22	1996
United States	0.78	9202.68	17 187.64	1996

sions 7 and 8). The coefficient estimates on β_1 and β_2 remain highly significant with the same signs and the estimated turning points do not change much. Furthermore, as a comparison, we run regressions (random-effects models) using mean income and its square value as regressors and controlling for the effects of the Gini coefficients (and its square term) as a measure of income inequality (regressions 1 and 2 in Table 4).[12] The estimation results show significant difference with those using median income in the regressions and the coefficient estimates in the case of NO_x become less significant.

Finally, because the median income used in the regressions is not directly observed and instead it is a created variable based on the mean income and the *m* ratio, potential measurement errors might exist.[13] To tackle this concern, we instrument for median income (and its square term) in the regressions and implement the instrumental variable (IV) estimation procedure in a random-effects estimation model. The instrumental variables are infant mortality rate (regressions 3 and 5 in Table 4) and life expectancy at birth (regressions 4 and 6

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[12] We thank an anonymous referee for making this suggestion.

[13] We thank an anonymous referee for pointing this out.

Table 4. Robustness checks

	Regressions using mean income		IV estimation				OECD subsample	
	SO_2	NO_x	SO_2		NO_x			
	(1) Random effects	(2) Random effects	(3) Random effects (Instrument: infant mortality)	(4) Random effects (Instrument: life expectancy)	(5) Random effects (Instrument: infant mortality)	(6) Random effects (Instrument: life expectancy)	SO_2 (7): Random effects	NO_x (8): Random effects
Log-median income (β_1)	–	–	11.91** (5.42)	16.49*** (3.97)	3.80* (2.04)	3.99** (1.61)	10.61*** (2.48)	3.29*** (1.04)
Log-median income squared (β_2)	–	–	–0.70** (0.30)	–0.94*** (0.22)	–0.21* (0.11)	–0.22** (0.09)	–0.60*** (0.13)	–0.17*** (0.06)
Log-mean income	12.98*** (4.01)	3.40* (1.75)	–	–	–	–	–	–
Log-mean income squared	–0.71*** (0.21)	–0.18* (0.09)	–	–	–	–	–	–
Gini coefficient	0.12 (0.12)	–0.01 (0.05)	–	–	–	–	–	–
Gini coefficient squared	–0.17E-02 (0.18E-02)	0.15E-03 (0.79E-03)	–	–	–	–	–	–
Other control variables	Included	Included	Included	Included	Included	Included	Included	Included
Constant	–59.51*** (18.10)	–15.47* (8.15)	–51.86** (23.90)	–72.66*** (17.61)	–16.86* (9.10)	–17.81** (7.25)	–47.18*** (11.12)	–15.13*** (4.72)
Turning point (in 2000 US dollars)	9328.65#	12 637.76#	4950.00	6630.00	8498.28	9675.86	6916.511	12 607.59
R^2	0.37	0.28	0.43	0.39	0.36	0.40	0.45	0.47
Number of observations	92	96	97	100	94	97	91	89

‘Log’ means natural logarithm of variable values. Control variables are population density, urbanization and their squares. ***, ** and * indicate that a coefficient estimate is significantly different from zero at the 1, 5 and 10% level, respectively. White heteroskedasticity-consistent standard errors are in parentheses except for instrumental variable (IV) estimation models. The IV for log-median income (and its quadratic term) in regressions (3) and (5) is the infant mortality rate, and the IV used in regressions (4) and (6) is life expectancy at birth. #, mean income; NO_x, oxides of nitrogen; SO_2, sulfur dioxide.

in Table 4), both of which are commonly used development indictors and are highly correlated with median income.[14] Data about these variables are obtained from the online WDI database compiled by the World Bank Group. Table 4 reports the IV estimation results for SO_2 (regressions 3 and 4) and NO_x (regressions 5 and 6), using infant mortality rate and life expectancy at birth, respectively, as IV for median income (and its quadratic term) in a random-effects model structure. The coefficients estimates on β_1 and β_2 remain significant and their signs are unchanged. Interestingly, the IV estimation results seem to suggest that the estimated turning point becomes lower for both SO_2 and NO_x after correcting for the potential measurement errors of the median income variable.

6. CONCLUSIONS

Using a variant of Andreoni and Levinson's (2001) model, the present paper investigates the effect of income distribution on the EKC and argues that the inverted U-shaped pattern of the pollution–income relationship is between median income and pollution level rather than between mean income and pollution level. The model suggests that a less skewed income distribution improves social efficiency in terms of the consumption and environmental quality of a society.

The results suggest that when examining the pollution–income relationship empirically, median income should be considered. Based on this argument, we estimate EKC for two major air pollutants using a panel data set that covers 36 countries over a 20-year period. Estimation results show that income distribution might be an important factor in the empirical estimation of EKC.

Two constraints on the interpretation of our results should be noted, and also suggest future research directions. First, the model in this paper assumes that majority rule is the mechanism that is used to make social choices. Although such a democratic system exists in a large number of countries, the degree of democracy varies among countries and different rules might apply. For instance, some countries are still ruled by dictators. Even in democracies, money matters in that rich people make campaign contributions and lobby for policies that will benefit them.[15] How such variation will affect the income/pollution relationship requires further study. The empirical component in this paper examines a sample that consists of mainly OECD countries (27 out of 36 countries); therefore, controlling for the effect of democratic institutions might not be a big issue. However, when other, less developed, countries are included in the estimation of an EKC, institutional factors should be properly controlled, among other variables.

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[14] One advantage of these instruments is that they contain information about the health-care input in a country that is not only related to the country's per capita GDP but also has an impact on income inequality.

[15] We thank an anonymous referee for pointing this out.

Second, empirical evidence that supports the existence of an EKC is found for only some forms of pollution, for example, air pollutants, but the evidence is mixed for other forms, such as water pollution (dissolved oxygen) and deforestation. Some environmental economists are still not convinced of the existence of an inverted U-shaped relationship between pollution and growth (e.g. Harbaugh *et al.*, 2002; Perman and Stern, 2003; and Deacon and Norman, 2006). One possible explanation is that the existing estimation models of EKC suffer from misspecification problems. Although the present paper points out one important factor (income distribution) that needs to be taken into account when estimating an EKC, more work is clearly needed along this line.

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APPENDIX: HEAD TAX REGIME

- *One-person case*

The basic setting is the same as that in Section 2 except that the individual pays a fixed amount of tax T instead of paying a proportional tax at the rate τ. The single consumer in this economy tries to solve the following maximization problem:

$$\begin{aligned}&\max_{T} U = C - \lambda P \\ &\textit{subject to } (2), (3) \textit{ and } E = T.\end{aligned} \tag{22}$$

The first-order condition is:

$$\frac{\partial U}{\partial T} = -1 + \lambda + \lambda (W - T)^{\alpha} \beta T^{\beta-1} - \lambda T^{\beta} \alpha (W - T)^{\alpha-1} = 0. \tag{23}$$

The second-order condition is:

$$\begin{aligned}\frac{\partial^2 U}{\partial T^2} = &-\lambda\beta\alpha (W - T)^{\alpha-1} \beta T^{\beta-1} - \lambda\beta(1-\beta)(W - T)^{\alpha} T^{\beta-2} \\ &- \lambda\beta\alpha T^{\beta-1}(W - T)^{\alpha-1} - \lambda\alpha(1-\alpha) T^{\beta}(W - T)^{\alpha-2} < 0.\end{aligned} \tag{24}$$

Total differentiation of the first-order condition yields:

$$\frac{\partial T^*}{\partial W} = \frac{\lambda\beta\alpha T^{\beta-1}(W - T)^{\alpha-1} + \lambda\alpha(1-\alpha) T^{\beta}(w - T)^{\alpha-2}}{-\partial^2 U/\partial T^{*2}} > 0. \tag{25}$$

Therefore, the consumer's preference is single peaked with respect to the individual tax level, and the optimal tax increases monotonically with respect to the consumer's income (endowment).

- *Multiple-consumer case*

There are N consumers in the economy. Each consumer pays a certain amount of tax. The amount of tax is decided by majority rule. Consumer *i's* problem can be summarized as:

$$\max_{T_i} U_i = C_i - \lambda P \qquad \text{subject to equation 2 and} \quad E = NT_i \tag{26}$$

$$C = C_i + C_{-i} = W_i + \sum_{j \neq i} W_j - NT_i \tag{27}$$

$$W_i = T_i + C_i. \tag{28}$$

The first-order condition is:

$$\begin{aligned}\frac{\partial U_i}{\partial T_i} &= -1+\lambda N-\lambda\alpha(W_i+W_{-i}-NT_i)^{\alpha-1}N^{\beta+1}T_i^{\beta}\\ &\quad+\lambda(W_i+W_{-i}-NT_i)^{\alpha}N^{\beta}\beta T_i^{\beta-1}=0. \end{aligned}\tag{29}$$

Checking the second-order condition, we have:

$$\begin{aligned}\frac{\partial^2 U_i}{\partial T_i^2} &= -\lambda\alpha(1-\alpha)N^{\beta+2}(W_i+W_{-i}-NT_i)^{\alpha-2}T_i^{\beta}\\ &\quad-\lambda\alpha\beta N^{\beta+1}(W_i+W_{-i}-NT_i)^{\alpha-1}T_i^{\beta-1}\\ &\quad-\lambda\alpha\beta N^{\beta+1}(W_i+W_{-i}-NT_i)^{\alpha-1}T_i^{\beta-1}\\ &\quad-\lambda\beta(1-\beta)N^{\beta}(W_i+W_{-i}-NT_i)^{\alpha}T_i^{\beta-1}\\ &<0. \end{aligned}\tag{30}$$

Total differentiation of the first-order condition gives:

$$\begin{aligned}\frac{\partial T_i^*}{\partial W_i} &= \frac{\lambda\alpha(1-\alpha)N^{\beta+1}(W_i+W_{-i}-NT_i)^{\alpha-2}T_i^{\beta}+\lambda\alpha\beta N^{\beta}(W_i+W_{-i}-NT_i)^{\alpha-1}T_i^{\beta-1}}{-\partial^2 U_i/\partial T_i^{*2}}\\ &>0. \end{aligned}\tag{31}$$

Therefore, each consumer's utility is single peaked with respect to the tax level, and the optimal tax level preferred by a consumer increases monotonically with income. These properties guarantee the existence of an MVE, and the individual with a median income is the decisive voter.

The social planner's problem in a multiple-consumer case is given by:

$$\max_T U=\sum_N U_i=\sum_N C_i-N\lambda P \qquad \text{subject to equation 2 and } E=NT \tag{32}$$

$$C=\sum_N W_i-NT. \tag{33}$$

The first-order condition is:

$$\frac{\partial U}{\partial T}=-N+N^2\lambda-N^{2+\beta}\lambda\alpha T^{\beta}\left(\sum_N W_i-NT\right)^{\alpha-1}+N^{1+\beta}\lambda\beta T^{\beta-1}\left(\sum_N W_i-NT\right)^{\alpha}=0. \tag{34}$$

Comparing equation 34 with the first-order condition of the individual consumer's optimization problem, one can see (under some weak assumptions, see Section 3 and footnote 3) that the socially efficient tax level will not be chosen by the decisive voter (the voter with a median income) and the MVE tax level is lower than the socially efficient tax level if $m < 1$, which implies underinvestment in pollution abatement.